1. Show that every entire function is given by a power series that converges locally uniformly on the complex plane. By an entire function we mean a function that is complex analytic on the entire plane.

2. For a given integer $j \in \mathbb{Z}$, find all entire functions $f$ that satisfy $|f(z)| \leq |e^z| |z - i|^j$ for each complex number $z \neq i + 1$.

3. In this question, $\Omega$ is a simply connected planar domain that is not the entire complex plane, and $z_0, z_1 \in \Omega$. Show that if $f$ and $g$ are two conformal maps of $\Omega$ that map $z_0$ to $z_1$, then $f = g$.

4. Compute the exact value of the integral $\int_0^{\infty} \frac{1}{x^{4n} + 1} \, dx$ for each positive integer $n$.

5. Let $D$ be the open disk $D = \{ z : |z - c| < \rho \}$, where $c, \rho \in \mathbb{R}$ with $0 < \rho < c$ and let $H$ denote the left half-plane $H = \{ z : \text{Re}(z) < 0 \}$. Find the image of the union $D \cup H$ under the mapping

$$z \mapsto \frac{z - a}{z + a}$$

where $a = \sqrt{c^2 - \rho^2} > 0$. 