The College of Arts & Sciences
Department of Mathematical Sciences
Colloquium

Professor Alexander Glazman

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Thursday, February 6, 2025
French Hall West, Room 4221
4:00-5:00pm

Random-cluster model on $\mathbb{Z}^2$ at the transition point

The random-cluster model is defined on subgraphs of $\mathbb{Z}^2$ and has two parameters: cluster-weight $q > 0$ and edge-probability $0 < p < 1$. It is classical that, for each $q \geq 1$, the model undergoes a percolation phase transition when $p = p_c(q)$. Beffara and Duminil-Copin in 2010 computed $p_c(q)$, and later works established the type of the phase transition: it is continuous when $1 \leq q \leq 4$ and discontinuous when $q > 4$. The former is characterized by Russo-Seymour-Welsh estimates, while the latter asserts non-uniqueness of the infinite-volume DLR/Gibbs measure.

In this talk we revisit both parts of this diagram. When $1 \leq q \leq 4$, we give a new proof of continuity that does not use parafermionic observable, nor Bethe Ansatz. When $q > 4$, we establish invariance principle under Dobrushin boundary conditions: the interface converges to the Brownian bridge. Both arguments rely on the Baxter-Kelland-Wu correspondence that relates the random-cluster model to a certain height function (six-vertex model). Remarkably, we obtain also some result when $q < 1$, though only at the self-dual point.

Joint works with Moritz Dober, Piet Lammers and Sebastien Ott.

Refreshments will be served 3:15-3:45 pm in the Faculty Lounge 4118 French Hall West

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