Statistics Qualifying Exam

12-4pm, Tuesday, August 19, 2025

Answer questions with showing all of your work.

This is closed-note/book. A calculator is allowed.

1. Let $X_1, \ldots, X_n$ be a random sample from a population with probablity density function (pdf)

$$f_X(x) = \begin{cases} \frac{1}{\theta}, & \text{if } 0 < x < \theta \\ 0, & \text{otherwise} \end{cases}$$

Let $X_{(1)} < \cdots < X_{(n)}$ be the order statistics. Show that $X_{(1)}/X_{(n)}$ and $X_{(n)}$ are independent random variables.

2. Let $X_1, \ldots, X_n$ be a random sample from $\mathcal{N}(\theta, 1)$.

   (a) Show that the unique UMVUE (Uniformly Minimum Variance Unbiased Estimator) of θ^2 is $\bar{X}^2 - \frac{1}{n}$.

   (b) Calculate the variance of $\bar{X}^2 - \frac{1}{n}$.
   Hint: It is known that the moment-generating function of a normal distribution $\mathcal{N}(\mu, \sigma^2)$ is

$$M_X(t) = \mathbb{E}[e^{tX}] = \exp\left(\mu t + \frac{1}{2}\sigma^2 t^2\right), \quad \text{for } t \in \mathbb{R} .$$

   (c) Is the estimator in (a) an efficient estimator? Clearly justify your answer.

3. Let $Y_1 < Y_2 < \cdots < Y_n$ denote the order statistics of a random sample of size n from a distribution that had pdf $f(x) = \frac{3x^2}{\theta^3}$, $0 < x < \theta$; zero elsewhere.

   (a) Show that $P(c < \frac{Y_n}{\theta} < 1) = 1 - c^{3n}$, where $0 < c < 1$.

   (b) If n is 4 and if the observed value of Y_4 is 2.3, what is a 95% confidence interval for θ?

4. Let $X_1, \ldots, X_n$ be a random sample from $N(\mu, \theta)$, $0 < \theta < \infty$ where μ is known.
   Let $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$. We know that $E(S^2) = \theta$.

   (a) What is the efficiency of S^2 ?

   (b) What is the MLE, $\hat{\theta}$, of θ ?

   (c) What is the asymptotic distribution of $\sqrt{n}(\hat{\theta} - \theta)$?

   (d) Find an exact Likelihood Ratio (LR) test for

$$\mathrm{H}_0 : \theta = \theta_0 \quad \text{v.s.} \quad \mathrm{H}_1 : \theta \neq \theta_0$$

   at significance level $\alpha \in (0, 1)$ where $\theta_0 \in (0, \infty)$ is fixed.

5. Suppose that we conduct the simple regression analysis with $n = 4$ observations $\{(y_1, x_1), (y_2, x_2), \cdots, (y_n, x_n)\}$ with the first-order model,

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \qquad \epsilon_i \overset{iid}{\sim} \mathcal{N}(0, \sigma^2).$$

The R code below is showing: (i) the vector y (response) and x (predictor); (ii) the calculated hat matrix $\boldsymbol{H} = \boldsymbol{X}(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T$; (iii) the linear model output from R `lm` function.

```
> y=c(4,5.5,6.2,7.8)
> x=c(2,3,4,5)
>
> X=cbind((rep(1,4)),(x))
>
> H=X%*%solve(t(X)%*%X)%*%t(X)
> H
[,1] [,2] [,3] [,4]
[1,]  0.7  0.4  0.1 -0.2
[2,]  0.4  0.3  0.2  0.1
[3,]  0.1  0.2  0.3  0.4
[4,] -0.2  0.1  0.4  0.7

> SLR=lm(y~x)
> summary(SLR)

Call:
lm(formula = y ~ x)

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   ??????     0.4455   3.681  0.06651 .
x             1.2100     0.1212   9.980  0.00989 **
---
Signif. codes:  0 '*' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2711 on 2 degrees of freedom
Multiple R-squared:  0.9803,    Adjusted R-squared:  0.9705
F-statistic:  99.6 on 1 and 2 DF,  p-value: 0.009892
```

(a) Write down the fitted model when the assumed model is $y = \beta_0 + \beta_1 x_1 + \epsilon$. Interpret the regression coefficient of x_1.

(b) Based on this output, find the residuals of the above regression.

(c) Given the above output find the estimated correlation between residual e_1 and e_4.

(d) Find the estimated variance of all 4 residuals $Var(e_i)$ for $i = 1, 2, 3, 4$. Are these variances equal?

(e) Based on this output, identify an influential point. Clearly state the metric used to identify the influential point.

6. The following example is adapted from Tryfos (1998, pp. 130-1). According to Tryfos: "Before construction begins, a bridge project goes through a number of stages of production, one of which is the design stage. This phase is composed of various activities, each of which contributes directly to the overall design time. . . . . In short, predicting the design time is helpful for budgeting and internal as well as external scheduling purposes."
   Information from 45 bridge projects was compiled for use in this study. The response and predictor variables are as follows:

   - Y = Time = design time in person-days
   - $X1$ = DArea = Deck area of bridge (000 sq ft)
   - $X2$ = CCost = Construction cost ($000)
   - $X3$ = Dwgs = Number of structural drawings
   - $X4$ = Length = Length of bridge (ft)
   - $X5$ = Spans = Number of spans

   A summary of the multiple linear regression is given in next page.

   Use t-values:$t_{0.025,40} = 2.021$, $t_{0.05,40} = 1.684$, $t_{0.025,5} = 2.7764$, $t_{0.05,5} = 2.1318$, $t_{0.025,39} = 2.022691$, $t_{0.05,39} = 1.684875$, $t_{0.025,6} = 2.44691$, $t_{0.05,6} = 1.94318$, $t_{0.025,41} = 2.019541$, $t_{0.05,41} = 1.682878$, $t_{0.025,42} = 2.018082$, $t_{0.05,42} = 1.681952$,

```
> MultiReg1=lm(formula = Y ~ X1 + X2 + X3 + X4 + X5)
>
> summary(MultiReg1)

Call:
lm(formula = Y ~ X1 + X2 + X3 + X4 + X5)

Residuals:
    Min      1Q  Median      3Q     Max
-81.816 -26.797  -9.674  24.882 180.443

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -34.83256   25.03837  -1.391    0.172
X1            0.24675    1.63170   0.151    0.881
X2           -0.02107    0.07143  -0.295    0.770
X3           19.68195    4.08583   4.817 2.23e-05 ***
X4            0.05186    0.10378   0.500    0.620
X5           15.50454   10.14243   1.529    0.134
---

Residual standard error: 55.31 on 39 degrees of freedom
Multiple R-squared:  0.7101,    Adjusted R-squared:  0.6729
F-statistic:  19.1 on 5 and 39 DF,  p-value: 1.435e-09
> anova(MultiReg1)
Analysis of Variance Table (Type I Sum of Squares)

Response: Y
          Df Sum Sq Mean Sq F value    Pr(>F)
X1         1 152044  152044 49.6918 1.834e-08 ***
X2         1  24248   24248  7.9248  0.007606 **
X3         1  93558   93558 30.5771 2.329e-06 ***
X4         1  15231   15231  4.9780  0.031492 *
X5         1   7150    7150  2.3369  0.134414
Residuals 39 119330    3060
---
```

(a) Is it worth including $X5$ in the model that already contains $X1$, $X2$, $X3$, and $X4$?

(b) Suppose you have decided in part (a) to drop $X5$ variable form the model. Is it worth including $X4$ in the model that already contains $X1$, $X2$ and $X3$? Explain. Use $\alpha = 0.05$, and give hypotheses, test statistic, which $F(a; df_1, df_2)$ you used, (the value or the range of) p-values, and conclusions, in order to get full credits.

(c) Assume modeling assumptions hold true. After fitting the regression model, the researchers concluded that the null hypothesis that $\beta_2 \geq 1$ was rejected at significance level $\alpha = 0.05$ against the alternative hypothesis that $\beta_2 < 1$. One of the researchers concluded: "At significance level $\alpha = 0.05$, we reject the null hypothesis." Do you agree with this assessment?

(d) We use the function regsubsets with nbest=4 in R to perform best-subset based model selection. Output is summarized below. rss denotes the sum of squares of errors/residuals (SSE). Based on the results, which model would you select to explain the design time in person-days using:

   i. A criteria.

   ii. Using a forward selection process.

   Explain all steps.

```
             model p   rsq    rss adjr2    cp   bic stderr
1               X3 2 0.635 150281 0.626  8.12 -37.7   59.1
2               X4 2 0.477 215346 0.465 29.38 -21.5   70.8
3               X5 2 0.466 219705 0.454 30.81 -20.6   71.5
4               X2 2 0.416 240166 0.403 37.49 -16.6   74.7
5            X3-X5 3 0.708 120095 0.694  0.25 -44.0   53.5
6            X3-X4 3 0.685 129468 0.670  3.31 -40.6   55.5
7            X1-X3 3 0.655 141833 0.639  7.35 -36.5   58.1
8            X2-X3 3 0.649 144491 0.632  8.22 -35.7   58.7
9         X3-X4-X5 4 0.709 119611 0.688  2.09 -40.4   54.0
10        X1-X3-X5 4 0.708 120094 0.687  2.25 -40.2   54.1
11        X2-X3-X5 4 0.708 120094 0.687  2.25 -40.2   54.1
12        X2-X3-X4 4 0.687 128763 0.664  5.08 -37.1   56.0
13     X2-X3-X4-X5 5 0.710 119400 0.681  4.02 -36.7   54.6
14     X1-X3-X4-X5 5 0.709 119596 0.680  4.09 -36.6   54.7
15     X1-X2-X3-X5 5 0.708 120093 0.679  4.25 -36.4   54.8
16     X1-X2-X3-X4 5 0.693 126480 0.662  6.34 -34.1   56.2
17  X1-X2-X3-X4-X5 6 0.710 119330 0.673  6.00 -32.9   55.3
```

7. A management information systems consultant conducted a small-scale study of five different daily summary reports (A: the greatest amount of details; B; C; D; E: the least amount of details with B, C, D in the middle with gradually decreased amount of details). Five sales executives were used in the study. Each was given one type of daily report for a month and then was asked to rate its helpfulness on a 25-point scale (0: no help; 25: extremely helpful). Over a five-month period, each executive received each type of report for one month according to the latin square design shown below. The helpfulness ratings follow. The SAS output for the analysis of this experiment is included below.

	Month				
Executive	March	April	May	June	July
Harrison	21(D)	8(A)	17(C)	9(B)	16(E)
Smith	5(A)	10(E)	3(B)	12(C)	15(D)
Carmichael	20(C)	10(B)	15(E)	22(D)	12(A)
Loeb	4(B)	17(D)	3(A)	9(E)	10(C)
Munch	17(E)	16(C)	20(D)	7(A)	11(B)

(a) Complete the ANOVA table above by filling in the missing values marked by "****" in the SAS output provided.

Table 1: ANOVA Table

source of variation	df	SS	MS	F	p-value
report					
executive (Row)					
month (Column)					
Error					
Total					

(b) Test weather or not the five types of reports differ in mean helpfulness; use significance level $\alpha = 0.01$. State the alternatives, decision rule, and conculsion. What is the p-value of the test?

(c) Analyze the effectiveness of the five types of reports by making all pairwise compuarisons among the treatment means. Use the Tukey procedure and a 95 percent family confidence coefficient. Summarize your findings.

SAS Output

Class Level Information

Class	**Levels**	**Values**
executive	5	Carmicha Harrison Loeb Munch Smith
month	5	April July June March May
report	5	A B C D E

Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr>F**
Model	****	758.48	****	****	<.0001
Error	****	19.28	****		
Corrected Total	****	777.76			

R-Square	**Coeff Var**	**Root MSE**	**rating Mean**
0.975211	10.25521	1.267544	12.36

Source	**DF**	**Type III SS**	**Mean Square**	**Pr > F**
executive	****	220.16	****	< 0.0001
month	****	10.96	****	0.2132
report	****	527.36	****	< 0.0001

Report Level	**N**	**Mean Rating**	**Std Dev**
A	5	7.00	3.39
B	5	7.40	3.65
C	5	15.00	4.00
D	5	19.00	2.92
E	5	13.40	3.65

8. The yield of a chemical process is being studied. The two most important variables are thought to be the pressure and the temperature. Three levels of each factor are selected, and a factorial experiment with two replicates is performed. Please see the attached SAS output (next page).

   (a) Write down the two-factor fixed effects model to analyze this experiment. Clearly label all the notations and specify all the appropriate assumptions and constraints in the model.

   (b) Estimate the difference between the treatment means $D = \mu_{11} - \mu_{13}$, where μ_{11} is the factor level mean with Temperature = 150 and Pressure = 200, and μ_{13} is the factor level mean with Temperature = 150 and Pressure = 230. Construct a 95% confidence interval for D. Interpret your results.

   (c) Carry out Tukey multiple comparisons if needed (use $\alpha = 0.05$) and draw conclusions about factor effects. If you find there is no need for multiple comparisons, justify your decision.

Class Level Information

Class	Levels	Values
temperature	3	150 160 170
pressure	3	200 215 230

Dependent Variable: yields

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	8	1.13777778	0.14222222	8.00	0.0026
Error	9	0.16000000	0.01777778		
Corrected Total	17	1.29777778			

R-Square	Coeff Var	Root MSE	yields Mean
0.876712	0.147474	0.133333	90.41111

Source	DF	Type III SS	Mean Square	F Value	Pr > F
temperature	2	0.30111111	0.15055556	8.47	0.0085
pressure	2	0.76777778	0.38388889	21.59	0.0004
temperature*pressure	4	0.06888889	0.01722222	0.97	0.4700

Level of temperature	N	------------yields----------- Mean	Std Dev
150	6	90.4166667	0.20412415
160	6	90.2500000	0.26645825
170	6	90.5666667	0.29439203

Level of pressure	N	------------yields----------- Mean	Std Dev
200	6	90.3666667	0.21602469
215	6	90.6833333	0.14719601
230	6	90.1833333	0.19407902

Level of temperature	Level of pressure	N	------------yields----------- Mean	Std Dev
150	200	2	90.3000000	0.14142136
150	215	2	90.6500000	0.07071068
150	230	2	90.3000000	0.14142136
160	200	2	90.2000000	0.14142136
160	215	2	90.5500000	0.07071068
160	230	2	90.0000000	0.14142136
170	200	2	90.6000000	0.14142136
170	215	2	90.8500000	0.07071068
170	230	2	90.2500000	0.21213203

TABLE II Percentage Points of the *t* Distribution

ν \ α	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.727	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.019	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551

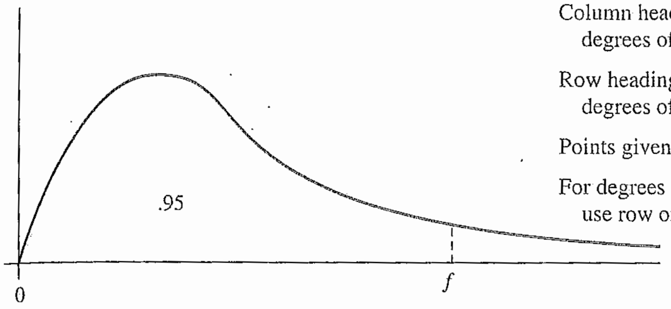
ν = Degrees of freedom.

TABLE V Percentage Points of the Studentized Range Statistic (*Continued*)

$q_{0.05}(p, f)$

f	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	18.1	26.7	32.8	37.2	40.5	43.1	45.4	47.3	49.1	50.6	51.9	53.2	54.3	55.4	56.3	57.2	58.0	58.8	59.6
2	6.09	8.28	9.80	10.89	11.73	12.43	13.03	13.54	13.99	14.39	14.75	15.08	15.38	15.65	15.91	16.14	16.36	16.57	16.77
3	4.50	5.88	6.83	7.51	8.04	8.47	8.85	9.18	9.46	9.72	9.95	10.16	10.35	10.52	10.69	10.84	10.98	11.12	11.24
4	3.93	5.00	5.76	6.31	6.73	7.06	7.35	7.60	7.83	8.03	8.21	8.37	8.52	8.67	8.80	8.92	9.03	9.14	9.24
5	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99	7.17	7.32	7.47	7.60	7.72	7.83	7.93	8.03	8.12	8.21
6	3.46	4.34	4.90	5.31	5.63	5.89	6.12	6.32	6.49	6.65	6.79	6.92	7.04	7.14	7.24	7.34	7.43	7.51	7.59
7	3.34	4.16	4.68	5.06	5.35	5.59	5.80	5.99	6.15	6.29	6.42	6.54	6.65	6.75	6.84	6.93	7.01	7.08	7.16
8	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92	6.05	6.18	6.29	6.39	6.48	6.57	6.65	6.73	6.80	6.87
9	3.20	3.95	4.42	4.76	5.02	5.24	5.43	5.60	5.74	5.87	5.98	6.09	6.19	6.28	6.36	6.44	6.51	6.58	6.65
10	3.15	3.88	4.33	4.66	4.91	5.12	5.30	5.46	5.60	5.72	5.83	5.93	6.03	6.12	6.20	6.27	6.34	6.41	6.47
11	3.11	3.82	4.26	4.58	4.82	5.03	5.20	5.35	5.49	5.61	5.71	5.81	5.90	5.98	6.06	6.14	6.20	6.27	6.33
12	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.40	5.51	5.61	5.71	5.80	5.88	5.95	6.02	6.09	6.15	6.21
13	3.06	3.73	4.15	4.46	4.69	4.88	5.05	5.19	5.32	5.43	5.53	5.63	5.71	5.79	5.86	5.93	6.00	6.06	6.11
14	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25	5.36	5.46	5.56	5.64	5.72	5.79	5.86	5.92	5.98	6.03
15	3.01	3.67	4.08	4.37	4.59	4.78	4.94	5.08	5.20	5.31	5.40	5.49	5.57	5.65	5.72	5.79	5.85	5.91	5.96
16	3.00	3.65	4.05	4.34	4.56	4.74	4.90	5.03	5.15	5.26	5.35	5.44	5.52	5.59	5.66	5.73	5.79	5.84	5.90
17	2.98	3.62	4.02	4.31	4.52	4.70	4.86	4.99	5.11	5.21	5.31	5.39	5.47	5.55	5.61	5.68	5.74	5.79	5.84
18	2.97	3.61	4.00	4.28	4.49	4.67	4.83	4.96	5.07	5.17	5.27	5.35	5.43	5.50	5.57	5.63	5.69	5.74	5.79
19	2.96	3.59	3.98	4.26	4.47	4.64	4.79	4.92	5.04	5.14	5.23	5.32	5.39	5.46	5.53	5.59	5.65	5.70	5.75
20	2.95	3.58	3.96	4.24	4.45	4.62	4.77	4.90	5.01	5.11	5.20	5.28	5.36	5.43	5.50	5.56	5.61	5.66	5.71
24	2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.81	4.92	5.01	5.10	5.18	5.25	5.32	5.38	5.44	5.50	5.55	5.59
30	2.89	3.48	3.84	4.11	4.30	4.46	4.60	4.72	4.83	4.92	5.00	5.08	5.15	5.21	5.27	5.33	5.38	5.43	5.48
40	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.74	4.82	4.90	4.98	5.05	5.11	5.17	5.22	5.27	5.32	5.36
60	2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.55	4.65	4.73	4.81	4.88	4.94	5.00	5.06	5.11	5.15	5.20	5.24

TABLE IX
F* distribution *(continued)



Column heading = numerator degrees of freedom

Row heading = denominator degrees of freedom

Points given are $f_{.05}$ points

For degrees of freedom > 120, use row or column 120

$P[F_{\gamma_1, \gamma_2} \leq f] = .95$

γ_2 \ γ_1	1	2	3	4	5	6	7	8
1	161.448	199.500	215.707	224.583	230.161	233.985	236.768	238.882
2	18.513	19.000	19.164	19.247	19.296	19.329	19.353	19.371
3	10.128	9.552	9.277	9.117	9.013	8.941	8.887	8.845
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726
8	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447
21	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266
31	4.160	3.305	2.911	2.679	2.523	2.409	2.323	2.255
32	4.149	3.295	2.901	2.668	2.512	2.399	2.313	2.244
33	4.139	3.285	2.892	2.659	2.503	2.389	2.303	2.235
34	4.130	3.276	2.883	2.650	2.494	2.380	2.294	2.225
35	4.121	3.267	2.874	2.641	2.485	2.372	2.285	2.217
36	4.113	3.259	2.866	2.634	2.477	2.364	2.277	2.209
37	4.105	3.252	2.859	2.626	2.470	2.356	2.270	2.201
38	4.098	3.245	2.852	2.619	2.463	2.349	2.262	2.194
39	4.091	3.238	2.845	2.612	2.456	2.342	2.255	2.187
40	4.085	3.232	2.839	2.606	2.449	2.336	2.249	2.180
50	4.034	3.183	2.790	2.557	2.400	2.286	2.199	2.130
60	4.001	3.150	2.758	2.525	2.368	2.254	2.167	2.097
120	3.9301	3.072	2.6681	2.447	2.290	2.175	2.087	2.016

TABLE IX
F* distribution *(continued)

γ_2 \ γ_1	9	10	11	12	13	14	15	16
1	240.543	241.881	242.983	243.905	244.689	245.363	245.949	246.462
2	19.385	19.396	19.405	19.412	19.419	19.424	19.429	19.433
3	8.812	8.786	8.763	8.745	8.729	8.715	8.703	8.692
4	5.999	5.964	5.936	5.912	5.891	5.873	5.858	5.844
5	4.772	4.735	4.704	4.678	4.655	4.636	4.619	4.604
6	4.099	4.060	4.027	4.000	3.976	3.956	3.938	3.922
7	3.677	3.637	3.603	3.575	3.550	3.529	3.511	3.494
8	3.388	3.347	3.313	3.284	3.259	3.237	3.218	3.202
9	3.179	3.137	3.102	3.073	3.048	3.025	3.006	2.989
10	3.020	2.978	2.943	2.913	2.887	2.865	2.845	2.828
11	2.896	2.854	2.818	2.788	2.761	2.739	2.719	2.701
12	2.796	2.753	2.717	2.687	2.660	2.637	2.617	2.599
13	2.714	2.671	2.635	2.604	2.577	2.554	2.533	2.515
14	2.646	2.602	2.566	2.534	2.507	2.484	2.463	2.445
15	2.588	2.544	2.507	2.475	2.448	2.424	2.403	2.385
16	2.538	2.494	2.456	2.425	2.397	2.373	2.352	2.333
17	2.494	2.450	2.413	2.381	2.353	2.329	2.308	2.289
18	2.456	2.412	2.374	2.342	2.314	2.290	2.269	2.250
19	2.423	2.378	2.340	2.308	2.280	2.256	2.234	2.215
20	2.393	2.348	2.310	2.278	2.250	2.225	2.203	2.184
21	2.366	2.321	2.283	2.250	2.222	2.197	2.176	2.156
22	2.342	2.297	2.259	2.226	2.198	2.173	2.151	2.131
23	2.320	2.275	2.236	2.204	2.175	2.150	2.128	2.109
24	2.300	2.255	2.216	2.183	2.155	2.130	2.108	2.088
25	2.282	2.236	2.198	2.165	2.136	2.111	2.089	2.069
26	2.265	2.220	2.181	2.148	2.119	2.094	2.072	2.052
27	2.250	2.204	2.166	2.132	2.103	2.078	2.056	2.036
28	2.236	2.190	2.151	2.118	2.089	2.064	2.041	2.021
29	2.223	2.177	2.138	2.105	2.075	2.050	2.027	2.007
30	2.211	2.165	2.126	2.092	2.063	2.037	2.015	1.995
31	2.199	2.153	2.114	2.080	2.051	2.026	2.003	1.983
32	2.189	2.142	2.103	2.070	2.040	2.015	1.992	1.972
33	2.179	2.133	2.093	2.060	2.030	2.004	1.982	1.961
34	2.170	2.123	2.084	2.050	2.021	1.995	1.972	1.952
35	2.161	2.114	2.075	2.041	2.012	1.986	1.963	1.942
36	2.153	2.106	2.067	2.033	2.003	1.977	1.954	1.934
37	2.145	2.098	2.059	2.025	1.995	1.969	1.946	1.926
38	2.138	2.091	2.051	2.017	1.988	1.962	1.939	1.918
39	2.131	2.084	2.044	2.010	1.981	1.954	1.931	1.911
40	2.124	2.077	2.038	2.003	1.974	1.948	1.924	1.904
50	2.073	2.026	1.986	1.952	1.921	1.895	1.871	1.850
60	2.040	1.993	1.952	1.917	1.887	1.860	1.836	1.815
120	1.959	1.910	1.869	1.834	1.803	1.775	1.750	1.728

16
246.462
19.433
8.692
5.844
4.604
3.922
3.494
3.202
2.989
2.828
2.701
2.599
2.515
2.445
2.385
2.333
2.289
2.250
2.215
2.184
2.156
2.131
2.109
2.088
2.069
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1.961
1.952
1.942
1.934
1.926
1.918
1.911
1.904
1.850
1.815
1.728

TABLE IX
F distribution (*continued*)

γ_2 \ γ_1	17	18	19	20	21	22	23	24
1	246.917	247.322	247.685	248.012	248.308	248.577	248.824	249.051
2	19.437	19.440	19.443	19.446	19.448	19.450	19.452	19.454
3	8.683	8.675	8.667	8.660	8.654	8.648	8.643	8.639
4	5.832	5.821	5.811	5.803	5.795	5.787	5.781	5.774
5	4.590	4.579	4.568	4.558	4.549	4.541	4.534	4.527
6	3.908	3.896	3.884	3.874	3.865	3.856	3.849	3.841
7	3.480	3.467	3.455	3.445	3.435	3.426	3.418	3.411
8	3.187	3.173	3.161	3.150	3.140	3.131	3.123	3.115
9	2.974	2.960	2.948	2.936	2.926	2.917	2.908	2.900
10	2.812	2.798	2.785	2.774	2.764	2.754	2.745	2.737
11	2.685	2.671	2.658	2.646	2.636	2.626	2.617	2.609
12	2.583	2.568	2.555	2.544	2.533	2.523	2.514	2.505
13	2.499	2.484	2.471	2.459	2.448	2.438	2.429	2.420
14	2.428	2.413	2.400	2.388	2.377	2.367	2.357	2.349
15	2.368	2.353	2.340	2.328	2.316	2.306	2.297	2.288
16	2.317	2.302	2.288	2.276	2.264	2.254	2.244	2.235
17	2.272	2.257	2.243	2.230	2.219	2.208	2.199	2.190
18	2.233	2.217	2.203	2.191	2.179	2.168	2.159	2.150
19	2.198	2.182	2.168	2.156	2.144	2.133	2.123	2.114
20	2.167	2.151	2.137	2.124	2.112	2.102	2.092	2.082
21	2.139	2.123	2.109	2.096	2.084	2.073	2.063	2.054
22	2.114	2.098	2.084	2.071	2.059	2.048	2.038	2.028
23	2.091	2.075	2.061	2.048	2.036	2.025	2.014	2.005
24	2.070	2.054	2.040	2.027	2.015	2.003	1.993	1.984
25	2.051	2.035	2.021	2.007	1.995	1.984	1.974	1.964
26	2.034	2.018	2.003	1.990	1.978	1.966	1.956	1.946
27	2.018	2.002	1.987	1.974	1.961	1.950	1.940	1.930
28	2.003	1.987	1.972	1.959	1.946	1.935	1.924	1.915
29	1.989	1.973	1.958	1.945	1.932	1.921	1.910	1.901
30	1.976	1.960	1.945	1.932	1.919	1.908	1.897	1.887
31	1.965	1.948	1.933	1.920	1.907	1.896	1.885	1.875
32	1.953	1.937	1.922	1.908	1.896	1.884	1.873	1.864
33	1.943	1.926	1.911	1.898	1.885	1.873	1.863	1.853
34	1.933	1.917	1.902	1.888	1.875	1.863	1.853	1.843
35	1.924	1.907	1.892	1.878	1.866	1.854	1.843	1.833
36	1.915	1.899	1.883	1.870	1.857	1.845	1.834	1.824
37	1.907	1.890	1.875	1.861	1.848	1.837	1.826	1.816
38	1.899	1.883	1.867	1.853	1.841	1.829	1.818	1.808
39	1.892	1.875	1.860	1.846	1.833	1.821	1.810	1.800
40	1.885	1.868	1.853	1.839	1.826	1.814	1.803	1.793
50	1.831	1.814	1.798	1.784	1.771	1.759	1.748	1.737
60	1.796	1.778	1.763	1.748	1.735	1.722	1.711	1.700
120	1.709	1.690	1.674	1.659	1.645	1.632	1.620	1.608