1. Let $\mathbf{A}$ be a non-singular square matrix. Prove that $\left(e^{\mathbf{A}t}\right)^{-1} = e^{-\mathbf{A}t}$.

2. Consider the equation
$$\dot{x} = rx\,(1-x) - px,$$
with $r, p > 0$.
   (a) Identify the equilibriums and determine their stability for representative values of r and p.
   (b) Draw the bifurcation diagram and identify the type of bifurcation as r varies.

3. Consider the ODE
$$\ddot{x} = x\left(1 - x^2\right).$$
   (a) Determine all equilibria and classify.
   (b) Sketch the phase portrait in detail.

4. Consider the system of ODEs
$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -kx - \varepsilon y^3\left(1 + x^2\right), \end{aligned}$$
where x represents the displacement of the spring and k is the spring constant with $k > 0$.
   (a) Explain why linear analysis at the origin is not useful to determine the stability of the origin.
   (b) Use an appropriate Liapunov function to determine the nature of the origin.

5. Find the general solution of the system
$$\dot{\boldsymbol{x}} = \begin{bmatrix} 1 & 2 & -1 \\ -2 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix} \boldsymbol{x}.$$