1. Let $V$ be a finite-dimensional vector space over $\mathbb{R}$, and $T : V \to V$ be linear. Suppose $T^3 v = v$ for all $v \in V$, and $T^2 v \neq v$ for all nonzero $v \in V$.
   
   a) Show that $T$ has no real eigenvalues.
   
   b) Show that $\dim V$ cannot be odd.

2. Prove Apollonius’s identity, using properties of the inner product: in a triangle in $\mathbb{R}^n$ with sides of length $a$, $b$, and $c$, let $d$ be the line segment from the midpoint of the side length $c$ to the opposite vertex. Then
   
   $$a^2 + b^2 = \frac{1}{2} c^2 + 2d^2.$$

3. Let $V$ and $W$ be vector spaces, with $S, T : V \to W$ linear. Let $U = \{v \in V : Sv = Tv\}$.
   
   a) Show that $U$ is a subspace of $V$.
   
   b) Suppose $S$ is injective. Show that $\dim U \leq \dim \text{range } (T)$.

4. Let $V$ be a finite-dimensional vector space, and let $S = \{v_1, \ldots, v_k\} \subseteq V$. Prove the following.
   
   a) If $S$ is linearly independent, then $S$ can be completed to a basis of $V$.
   
   b) If $S$ spans $V$, then $S$ contains a basis of $V$.

5. Let $V$ and $W$ be finite-dimensional vector spaces of dimensions $n$ and $m$, respectively, and write $\mathcal{L}(V, W)$ for the set of all linear maps from $V$ to $W$. Prove that $\mathcal{L}(V, W)$ is a vector space. What is its dimension?