1. Let $(X_n)_{n\geq 1}$ be a sequence of independent and identically distributed random variables with
$$\mathbb{P}(X_n = 1) = \mathbb{P}(X_n = -1) = \frac{1}{2}.$$
Find the set of all real numbers $\beta \in \mathbb{R}$ such that
$$\frac{X_1 + \cdots + X_n}{n^\beta}$$
converges in probability to zero as $n \to \infty$.

2. Fix $\alpha > 0$. Let $(X_n)_{n\in\mathbb{N}}$ be independent and identically distributed random variables with
$$\mathbb{P}(X_1 > x) = x^{-\alpha}, \quad x \geq 1.$$
Define
$$Y_n = \frac{X_n}{n}, \quad n \in \mathbb{N}.$$
   (a) Find all values of $\alpha > 0$ such that $Y_n \to 0$ in probability as $n \to \infty$.
   (b) Find all values of $\alpha > 0$ such that $Y_n \to 0$ almost surely as $n \to \infty$.
   (c) Suppose the independence assumption is removed, i.e. $(X_n)_{n\in\mathbb{N}}$ are only identically distributed (not necessarily independent). How do your answers in parts (a) and (b) change?

3. Let $(X_n)_{n\geq 1}$ be independent random variables such that for each $n \geq 1$, $X_n$ has density $f_{X_n}(t) = ne^{-nt}\mathbb{1}_{t\geq 0}$.
   (a) Compute $\mathbb{P}(\min(X_1, \ldots, X_n) > t)$ for $t > 0$.
   (b) Find a constant $\beta > 0$ such that $n^\beta \cdot \min(X_1, \ldots, X_n)$ converges in distribution to a non-degenerate random variable as $n \to \infty$, and identify the limiting distribution.

4. Let $(X_n)_{n\in\mathbb{N}}$ be i.i.d. non-negative random variables.
   (a) Suppose $\mathbb{P}(X_1 > x) = e^{-x}$ for $x > 0$. Show that
$$\mathbb{P}(X_n > p \log n \text{ i.o.}) = 0, \quad \text{for all } p > 1. \tag{1}$$
   (b) Suppose instead the following weaker assumption on $X_1$ holds:
$$\lim_{x\to\infty} \frac{1}{x} \log \mathbb{P}(X_1 > x) = -1.$$
   Prove that (1) is still true.