

Complex Analysis Prelim Exam
UC Department of Math
Fall 25

Only answer 5 out of the 6 questions. Choose and indicate clearly which 5 you have chosen.

1. Find all the real numbers a for which the function $u(x, y) = \cos(ay) e^{x/3}$ is the real part of a complex analytic function on \mathbb{C} .

2. Calculate the value of the integral

$$\int_0^\infty \frac{\cos(5x)}{(x^2 + 1)(x^2 + 4)} dx.$$

3. Prove that the only analytic map from \mathbb{C} into $H := \{z \in \mathbb{C} : \operatorname{Re}(z) > 0\}$ must be constant.

4. Determine the number of zeroes of the function $f(z) = 2e^{2z} + z^2 + 4z + 3$ in the left half-plane. *Hint, consider the curve given by the half circle $\{z = Re^{i\theta} \text{ with } \pi/2 \leq \theta \leq 3\pi/2\} \cup [-iR, iR]$.*

5. Let $B_R(0) := \{z \in \mathbb{C} : |z| < R\}$. Suppose f is an entire function, $k \geq 1$ is an integer and for some $C > 1$, f satisfies

$$\sup_{B_R(0)} |f| \leq CR^k \text{ for all } R > 0.$$

(a) Prove that f is a polynomial of degree at most k .

(b) Prove that $f(z) = cz^k$ for some constant $c \in \mathbb{C}$.

6. Let $F(z) = \frac{z+1}{z-1}$. Let $L_c = \{z \in \mathbb{C} : \operatorname{Re}(z) = c\}$. Find the image of L_c under F for every $c \in \mathbb{R}$.