Statistics Qualifying Exam

September 5, 2006

Name :

1. Let $X_1, \ldots, X_n$ be i.i.d. $N(\mu, 1)$, $\mu \in (-\infty, \infty)$. We wish to estimate the parametric function $\tau(\mu) = \mu^2$ unbiasedly.

   (a) Based on X_1 and X_2, produce an unbiased estimator of $\tau(\mu) = \mu^2$.

   (b) Obtain the uniformly minimum variance unbiased estimator (UMVUE) for $\tau(\mu) = \mu^2$.

2. Let X_1 and X_2 be the random variables with joint probability mass function (p.m.f.)

$$p(x_1, x_2) = \theta_1\theta_2(1-\theta_1)^{x_1}(1-\theta_2)^{x_2}$$

   for $x_1 = 0, 1, 2, \ldots$ and $x_2 = 0, 1, 2, \ldots$. What is the p.m.f. of $X_1 - X_2$?

3. Let $X_1, \ldots, X_n$ be i.i.d. random variables with p.d.f

$$f(x;\beta) = \frac{2}{\beta}e^{-\frac{x^2}{\beta}}, \quad x > 0, \quad \beta > 0.$$

   (a) Find the maximum likelihood estimator (MLE) of β.

   (b) Find the maximum likelihood estimator (MLE) of $\sqrt{\beta}$.

4. Let X_1 and X_2 be a random sample of size $n = 2$ from a poisson distribution with mean λ. Reject the simple null hypothesis $H_0 : \lambda = 0.5$ and accept $H_1 : \lambda > 0.5$ if the observed sum $\sum_{i=1}^{2} x_i \geq 2$.
   Compute the type I error α of the test. Assume the alternative value of $\lambda = 0.6$, compute type II error. Find the power of the test.

5. Find the moment generating function of $X \sim f(x) = 1$, where $0 < x < 1$, and thereby confirm that $E(X) = 1/2$ and $V(X) = 1/12$.

6. (a) Assume the usual model for a one-way ANOVA with 4 groups and 6 observations per group. Find the usual estimate σ^2 if SSE = 60.

   (b) For a one-way ANOVA with 3 groups and 4 observations per group, give the degrees of freedom for the F statistic that is used to compare the group means.

   (c) Suppose that MSE = 25 in a one-way ANOVA with 3 groups and sample sizes $J_1 = 10$, $J_2 = 20$ and $J_3 = 15$. Give the standard deviation of the estimate of the contrast that compares the average of the means of the first two groups with the mean of the third group.

7. In a two-factor ANOVA, factor A has 3 levels and factor B has 4 levels. There are 3 observations per treatment.

   (a) Give the degrees of freedom for each of the following(as they would appear in the ANOVA tables):
   (i) factor A
   (ii) factor B
   (iii) interaction A*B
   (iv) model
   (v) error

   (b) SSA = 60 and SSE = 120. Calculate the F-statistics for testing for a factor A main effect, and give the degrees of freedom for that test.

   (c) Will the type I and type III sums of squares be the same or different in this analysis? Give a clear answer and a brief explanation.

8. Let $y_i = \beta_1 x_{1i} + \beta_2 x_{2i} + \epsilon_i$, $i = 1, \ldots, n$ $(n > 3)$ where x's are known constants and $\epsilon_1, \epsilon_2, \ldots, \epsilon_n$ are i.i.d. $N(0, \sigma^2)$.

   (a) Use the methods of Least Square to estimate β_1 and β_2.

   (b) Show that the estimators of β_1 and β_2 are unbiased

   (c) Find the standard error of the estimators of β_1 and β_2.

9. Consider the linear model

$$\begin{aligned} Y_1 &= \beta_1 + \beta_2 + \beta_3 + \epsilon_1 \\ Y_2 &= \beta_1 + \beta_3 + \epsilon_2 \\ Y_3 &= \beta_2 + \beta_3 + \epsilon_3 \end{aligned}$$

   where $\epsilon_1, \epsilon_2, \epsilon_3$ i.i.d. $N(0, \sigma^2)$.

   (a) Express in the form $\mathbf{y} = \mathbf{X}\beta + \epsilon$ where $\beta = (\beta_1, \beta_2, \beta_3)'$.

   (b) Estimate $\beta_1 - 2\beta_2 + \beta_3$ and obtain its variance.