1. Let

\[ f(z) = y - 2xy + i(-x + x^2 - y^2) + z^2 \]

where \( z = x + iy \). For what values of \( z \) does \( f'(z) \) exist?

2. Use the Residue Theorem to compute

\[ I = \oint_{|z|=2} \frac{z+2}{z(z+1)} \, dz, \]

where the contour of integration is oriented counter-clockwise.

3. Consider the horizontal strip \( S = \{ z : -\frac{\pi}{2} < \text{Im}(z) < \frac{\pi}{2} \} \). Find all the conformal maps that maps \( S \) to the open unit disk and map 0 to 0 with \( f'(0) > 0 \). For partial credit find one such mapping.

4. Show that \( e^z - (5z^2 + 1) = 0 \) has exactly two roots in the open unit disk \( |z| < 1 \).

5. Show that an entire function \( f \) on \( \mathbb{C} \) satisfying \( |f(z)| \leq \sqrt{1 + |z|} \) for all \( z \in \mathbb{C} \) is constant.