1. Prove that a non-constant entire function \( f \) cannot satisfy both \( f(z + 1) = f(z) \) and \( f(z + i) = f(z) \) for all \( z \in \mathbb{C} \).

2. Suppose that \( f : \mathbb{D} \to \mathbb{C} \) is holomorphic and that \( f \) is injective in some annulus \( A = \{ z : r < |z| < 1 \} \), with \( r > 0 \). Show that \( f \) is injective in \( \mathbb{D} \).

3. Find a Laurent series expansion of \( f(z) = \frac{1}{(z-1)(z-2)} \) that is valid in the annulus \( 1 < |z| < 2 \).

4. Use the residue theorem to compute
\[
\int_{-\infty}^{+\infty} \frac{1}{(x^2 + 1)^2} \, dx.
\]

5. Find a conformal map that maps the region \( \{ z \in \mathbb{C} : Re(z) > 0, 0 < Im(z) < 4\pi \} \) to the half-disk \( \{ z \in \mathbb{C} : |z| < 1, Im(z) > 0 \} \) and maps \( 1 + i \) to \( i/2 \). (Hint: Think about exponential functions)

6. Suppose \( g : \mathbb{C} \to \mathbb{C} \) is a holomorphic function, \( k, n \) are integers and
\[
|g^{(n)}(z)| \leq 3|z|^k \text{ when } |z| \geq 1,
\]
show \( g \) is a polynomial and estimate the degree of \( g \) in terms of \( k, n \).