(1) Suppose \((f_n)\) is a sequence of entire functions that converges locally uniformly in \(\mathbb{C}\) to a polynomial \(p\) with \(m = \deg p > 0\). Prove that for all sufficiently large \(n\), the number of zeroes of \(f_n\) (counted according to multiplicity) is at least \(m\).

(2) Suppose \(f\) is a meromorphic function (recall this is a function that is holomorphic except on a set of isolated points that are poles) on \(\mathbb{C}\) with \(\lim_{z \to \infty} f(z) = 0\).

(a) Show that \(f\) has finitely many poles in \(\mathbb{C}\).

(b) Use part (a) to show that \(f\) is a rational function. *Hint. Make adaptations to \(f\) to turn it into entire function \(g\). Think carefully about the growth of \(g\) at \(\infty\). What does this say about \(g\)*?

(3) Let \(T\) be the Möbius transformation that maps \(i, -i, \infty\) to \(\omega, \bar{\omega}, 1\) respectively, where \(\omega := e^{2\pi i/3}\). Determine the following images:

(a) \(T(\{iy \mid y \in \mathbb{R}\})\),

(b) \(T(\mathbb{H})\) where \(\mathbb{H} = \{z \in \mathbb{C} \mid \Re(z) > 0\}\),

(c) \(T(\mathbb{D})\) where \(\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}\).

(4) Let \(\Gamma\) be a piecewise smooth closed curve in \(\mathbb{C} \setminus \mathbb{Z}\). Calculate

\[
\int_{\Gamma} \frac{dz}{z(z^2 - 1)}.
\]

*Hint: there are different cases to consider depending on what the curve \(\Gamma\) does.*