Show all essential work.

1. A space is *totally disconnected* if its only connected subspaces are one-point sets. Show that if \( X \) has the discrete topology, then \( X \) is totally disconnected. Does the converse hold?

2. Let \( \mathbb{Z}^+ \) denote the set of positive integers; and \( X = \{ x \in \mathbb{Z}^+ : x \geq 2 \} \), together with the topology generated by the subbasis \( \{ U_n : n \geq 2 \} \), where \( U_n = \{ x \in \mathbb{Z}^+ : x \text{ divides } n \} \).
   
   (1) Is \( X \) Hausdor? 
   (2) Is \( X \) connected? Path connected? 
   (3) Is \( X \) locally compact? Compact?

3. Let \( X \) be a Hausdor space. Show that following are equivalent.
   
   (1) \( X \) is a compact space. 
   (2) For every topological space \( Y \) the projection \( p : X \times Y \to Y \) is closed. 
   (3) For every normal topological space \( Y \) the projection \( p : X \times Y \to Y \) is closed.

4. Prove: If \( X, Y \) are connected topological space with proper subsets \( A \subset X, B \subset Y \), then \( (X \times Y) \setminus (A \times B) \) is connected.

5. On fundamental groups and homeomorphisms.
   
   (1) Define fundamental group and give 3 distinct examples. Explain. 
   (2) Which pairs are homeomorphic? Prove why and/or why not. 
      \( \mathbb{R}, \ \mathbb{R}^2, \ \mathbb{R}^3 \)
      
      Hints: Remove a point. And for the latter two, think also homotopy and its connection to fundamental group (e.g., simply connectedness (i.e., loops homotopic to constant), etc.).