1. Suppose that $f''$ is continuous on $[0, \pi]$ and that

$$\int_0^\pi [f(x) + f''(x)] \sin x \, dx = 2.$$ 

Given that $f(\pi) = 3$, compute $f(0)$. 


2. Let $A(t)$ be the area under the curve $y = \sin(x^2), 0 \leq x \leq t$. Let $B(t)$ be the area of the triangle with the vertices $(0, 0)$, $(t, \sin(t^2))$, and $(t, 0)$. Find $\lim_{t \to 0^+} \frac{A(t)}{B(t)}$. 
3. A right triangle whose three sides have lengths 3, 4, and 5 ft is rotated about its hypotenuse. Compute the area of the resulting surface of revolution.
4. Let \( f(x) = \int_1^x \frac{\ln t}{1 + t} \, dt \) for \( x > 0 \). Find a formula for \( f(x) + f\left(\frac{1}{x}\right) \) that does not involve integrals.
5. Find the area of the region enclosed by the polar curve \( r(\theta) = (1 + \sin \theta)^{1/4} \), \( 0 \leq \theta \leq 2\pi \).

(Hint for your integral: \( 1 = \sin^2(\theta/2) + \cos^2(\theta/2) \).)
6. (a) Find the Maclaurin series for $\cos^2(x)$ and state its radius of convergence. (Hint: no need to square a power series.)

(b) Use the fact that

\[
\int_{-\infty}^{\infty} e^{-x^2} \, dx = \sqrt{\pi} \quad \text{and} \quad \int_{-\infty}^{\infty} x^{2n} e^{-x^2} \, dx = \frac{(2n-1)!}{2^{2n-1}(n-1)!} \sqrt{\pi}, \ n \geq 1,
\]

together with your result for part (a), to show that

\[
\int_{-\infty}^{\infty} e^{-x^2} \cos^2(x) \, dx = \frac{(1 + e)\sqrt{\pi}}{2e}.
\]
7. Let $f$ be a continuous function on the interval $[0, 1]$ such that $\int_0^1 f(t) \, dt = 0$. Show that

$$\int_0^1 e^{af(t)} \, dt \geq 1$$

for any real number $a$.

(Hint: treat the left-hand side of this inequality as a function of $a$ and examine its derivative(s), in particular at zero. Assume that you can differentiate under the integral sign.)