UNIVERSITY OF Cincinnati

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GRADUATE PROGRAM *QUALIFYING EXAM TOPICS*

Advanced Calculus Topics

Number Fields: basic axioms and operations for real numbers; least upper bound property (completeness); sequences

Topology: open, closed, compact, and connected sets in $\mathbb{R}^n$ and their properties; Bolzano-Weierstrass and Heine-Borel theorems; cluster points of an infinite set

Metric Spaces: bounded and unbounded sets; convergent sequences; Cauchy sequences; completeness

Series: tests for convergence and absolute convergence

Functions: limits at cluster points; continuity via limits, ε-δ, and pre-images of open or closed sets; upper and lower limits (aka, limit inferior and limit superior); continuity and compactness (Extreme Values theorem); continuity and connectedness (Intermediate Value theorem);

Differentiation I: limits and derivatives of functions on the real line; properties of derivatives (product and quotient rules, chain rule, Rolles' theorem, Mean Value theorem, l'Hôpital's rule); higher order derivatives

Function Sequences: uniform vs pointwise convergence; equicontinuity and the Arzela-Ascoli theorem; Weierstrass M-test; sequence of step functions converging uniformly to continuous functions on compact interval

Power Series: radius of convergence; Taylor's series and theorem; analytic functions vs smooth functions

Differentiation II: mappings between space domains and linear mappings; differentiability at a point; derivatives vs partial derivatives; differentiability of functions whose partial derivatives are continuous at a point; Chain Rule; the Inverse and Implicit Function theorems

Integration: Riemann integrals on intervals in the real line; upper and lower Riemann sums; integrability of step functions; Riemann integrability of uniform limits of Riemann integrable functions; integrability of continuous functions; properties of integrals; change of variables and the Jacobian

Advanced Calculus References

(1) Apostol, *Mathematical Analysis*, 2nd edition, 1974
(2) Dence and Dence, *Advanced Calculus: A Transition to Analysis*, 2010
(3) Folland, *Advanced Calculus*, 2002
(4) Rosenlicht, *Introduction to Analysis*, 1986
(5) Rudin, *Principles of Mathematical Analysis*, 3rd edition, 1976

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Linear Algebra Topics

Vector Spaces: fields, subspaces, isomorphisms, linear independence, bases, dimension

Linear Mappings: kernel, image, range, composition, inverses, similarities, projections

Matrices: homogenous and nonhomogenous linear equations, Gaussian elimination, null space, column and row spaces, rank, Rank-Nullity theorem, trace, matrix representative of a linear map, similarity of matrices, positive definite matrices, inverse of a matrix

Orthogonality: norms and inner products, Schwarz and triangle inequalities, orthogonality, orthonormal bases, Gram-Schmidt procedure

Determinants: definition, properties, signed volume, Cramer's rule, inverse of a matrix

Symmetry: symmetric, Hermitian, and unitary matrices

Spectral Theory: eigenvectors and eigenvalues, characteristic polynomial, diagonalization of symmetric and Hermitian matrices, generalized eigenvectors and Jordan canonical form

Linear Algebra References

(1) Artin, *Algebra*, 2nd edition, 2010
(2) Lang, *Linear Algebra*, 3rd edition, 2010