Differentiability of Lipschitz Functions and Curves in Carnot Groups

A function is Lipschitz if it increases distances by at most a constant factor. Rademacher’s theorem states that each Lipschitz function between Euclidean spaces is differentiable outside a set of measure zero. We ask if the converse holds: if N has measure zero, does there exist a Lipschitz function differentiable at no point of N? Another setting in which a Rademacher type theorem holds is that of Carnot groups. Carnot groups have strong geometric structure, including translations and dilations, but a distinguished family of “horizontal” directions. We ask whether general horizontal curves can be approximated by smooth horizontal curves.