Thurston maps, quasispheres, and matings

We consider fractal spheres, i.e., metric spaces that are homeomorphic to $S^2$. Such spheres arise in particular in geometric group theory (as boundaries at infinity of Gromov hyperbolic groups), in probability (i.e., the “Brownian map”), as well as in dynamics (by a snowflake type construction). An important question is whether such spheres are “quasispheres”, i.e., quasisymmetric images of the standard sphere. “Cannon's conjecture” stipulates that all spheres arising in the group setting are quasispheres. Similarly, certain branched covering maps of the sphere “are” rational maps if and only if a certain metric sphere is a quasisphere. The spheres in the three different settings can all be described as Peano curves. Alternatively they are obtained by “mating” two trees together.